General Physics: Electricity & Magnetism

Shuai Dong

Faraday cage

Chapter 2: Conductor & dielectrics

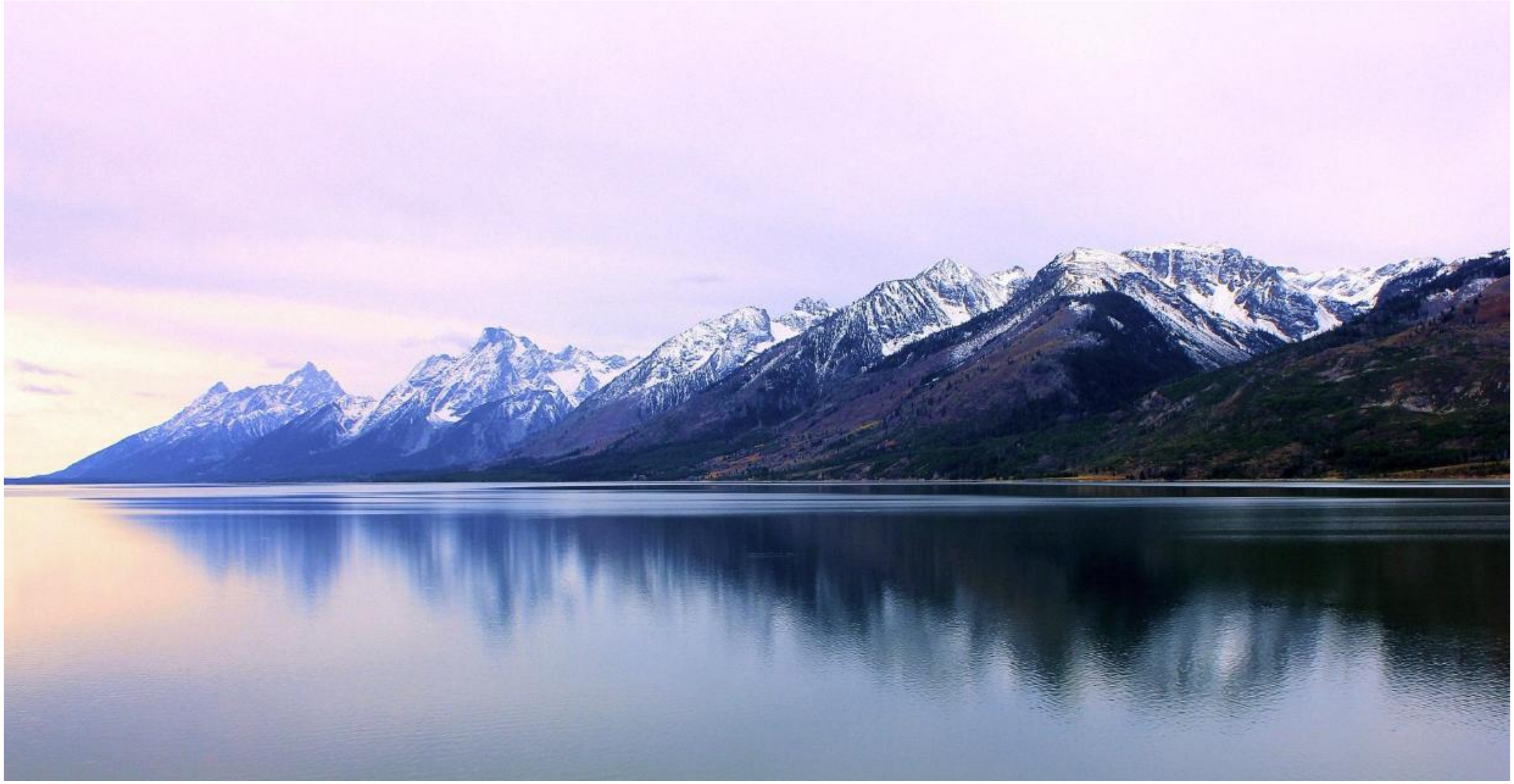
1. Conductors
2. Capacitance & capacitors
3. Dielectrics
4. Energy of electrostatic fields

Chapter 2: Homework

1. 9-2, 9-3, 9-5, 9-11, 9-14
2. Others in the ppt

2.1 Conductors

Equilibrium: stationary state; time-independent

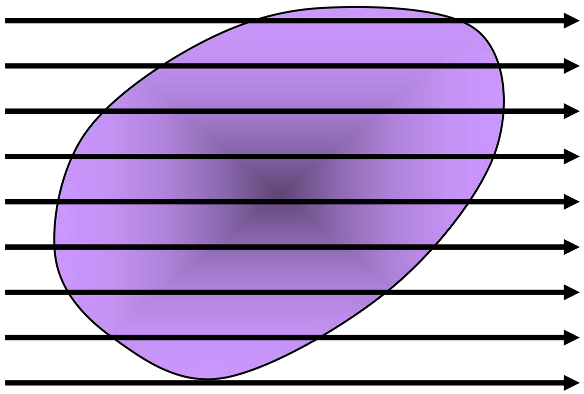


Mountain: stationary

Water: stationary?

Current: stationary?

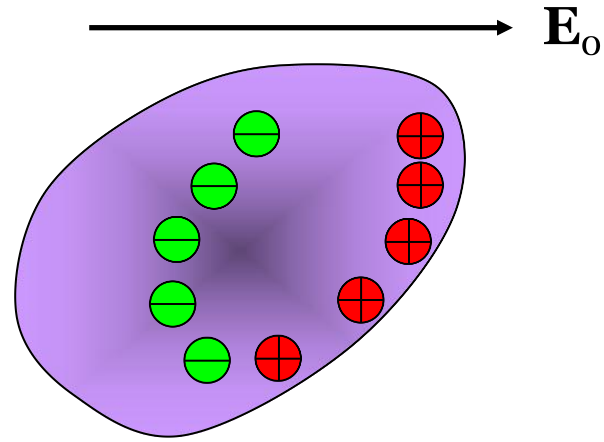
2.1 Conductors

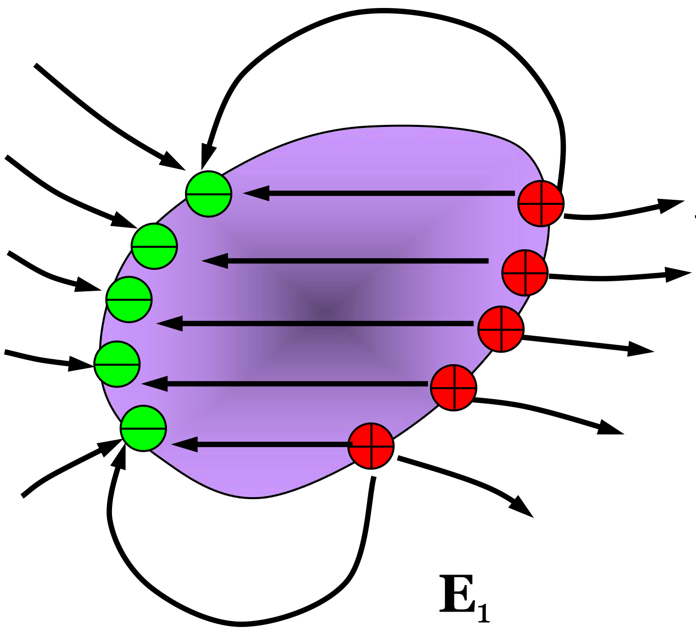


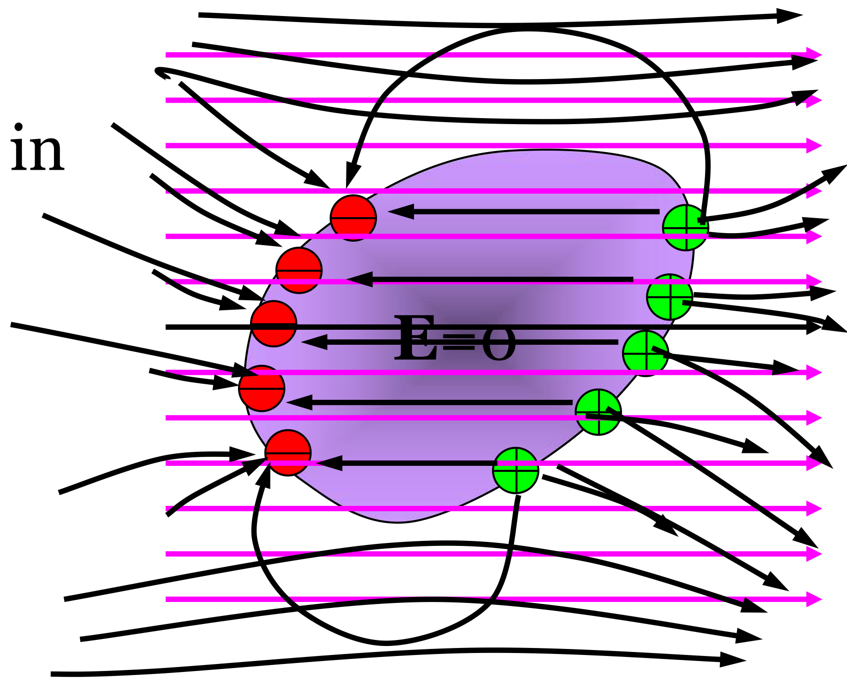
$\mathbf{E_0}$

Conclusion 1:

In an ideal conductor, the electrostatic field $\mathbf{E}=\mathbf{E_0}+\mathbf{E_1}=0$ in the equilibrium condition

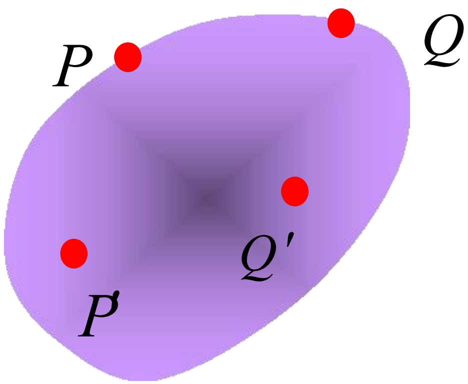






2.1 Conductors

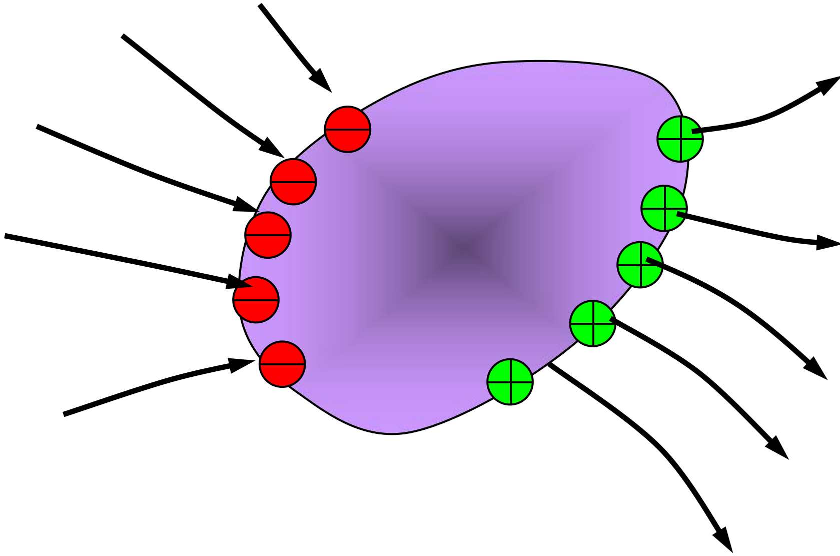
Conclusion 2: a conductor is an equipotential (isopotential) body



$$\Delta U_{PQ}=\int_P^Q \mathbf{E}\cdot d\mathbf{l}=0$$

Conclusion 3: E||n everywhere

Note: the pre-condition for electrostatic screening is that there are enough mobile carriers. Is it always satisfied?



2.1 Conductors

Example: to estimate the electrostatic screening length.

Surface density σ=1 e/a^2 a=4 Angstrom

$E=\sigma/\varepsilon_0=1.6\times10^{-19}/(4\times10^{-10})^2/8.85\times10^{-12}=1.1\times10^{11}$ N/C

case 1: Typical metal $\rho=10^{22}$ e/cm^3

$d=\sigma/\rho=1/(4\times10^{-10})^2/[10^{22}/(10^{-2})^3]=6.25\times10^{-10}$ m

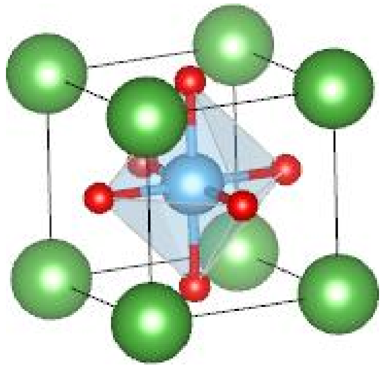
=6.25 A=1.5625 a

case 2: Intrinsic semiconductor Si $\rho=1.5\times10^{10}$ e/cm^3

d=417 m

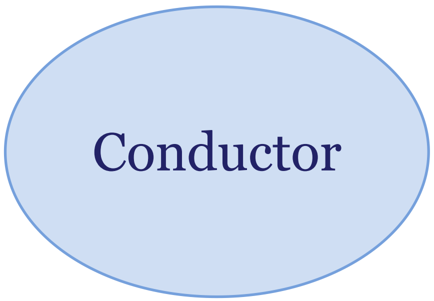
case 3: Doped semiconductor Si $\rho=10^{18}$ e/cm^3

d=6.25 μm=15625 a



2.1 Conductors

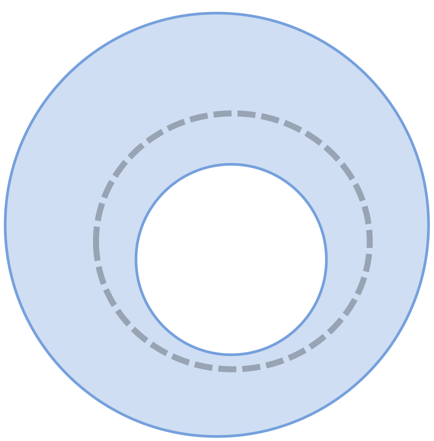
Conclusion 4: no net charge in a conductor



$$\Phi_e = \oint_S \mathbf{E}\cdot d\mathbf{S} = \frac{1}{\varepsilon_0}\sum_{i=1}^{n} q_i^{in}$$

$\nabla\cdot\mathbf{E} = \rho/\varepsilon_0$ → ρ=0 everywhere

Net charge can only appear at the surface of a conductor



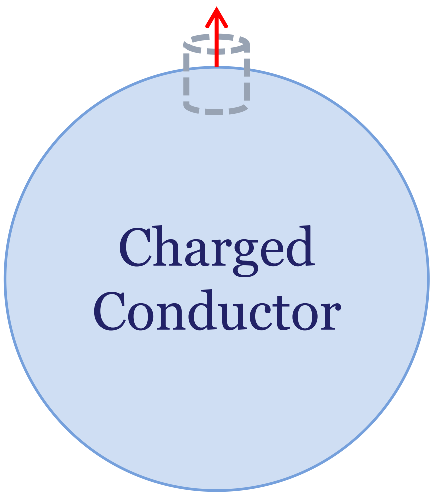
How about a conductor with a cavity?

net charge in the cavity = net charge at the inner surface

Applications: 1. Faraday cylinder

2. Van de Graaff generator

2.1 Conductors



Surface charge density $\sigma(\mathbf{r})$

$\mathbf{E}$ is perpendicular to the surface

$$\Phi = \int_{dS} \mathbf{E} \cdot d\mathbf{S} + \int_{S-dS} \mathbf{E} \cdot d\mathbf{S}$$

$$= \int_{dS} \mathbf{E} \cdot d\mathbf{S} + 0$$

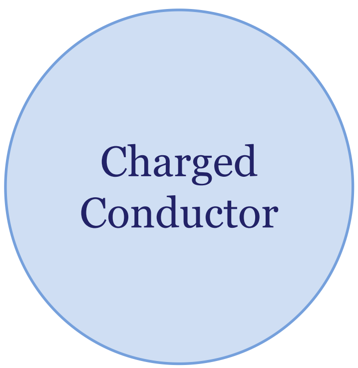
$$= EdS = dq/\varepsilon_0 = \sigma dS/\varepsilon_0$$

$$\mathbf{E} = \sigma(\mathbf{r})/\varepsilon_0$$

For comparison, $\mathbf{E} = \sigma(\mathbf{r})/2\varepsilon_0$ for infinite plate because there are two surfaces.

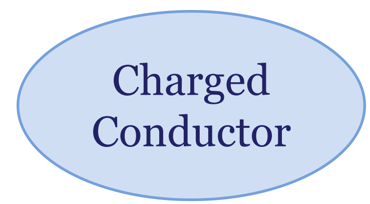
2.1 Conductors

For an isolated conductor, the surface charge density may be complex in general, but qualitatively it is in proportional to the local curvature



Sphere: uniform curvature, uniform σ

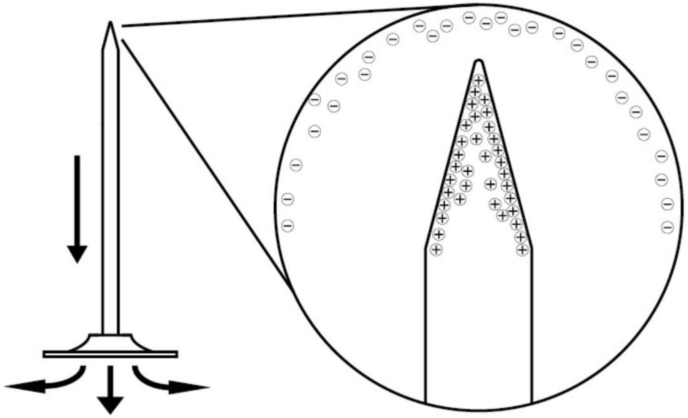
Ellipsoid: larger σ at two ends



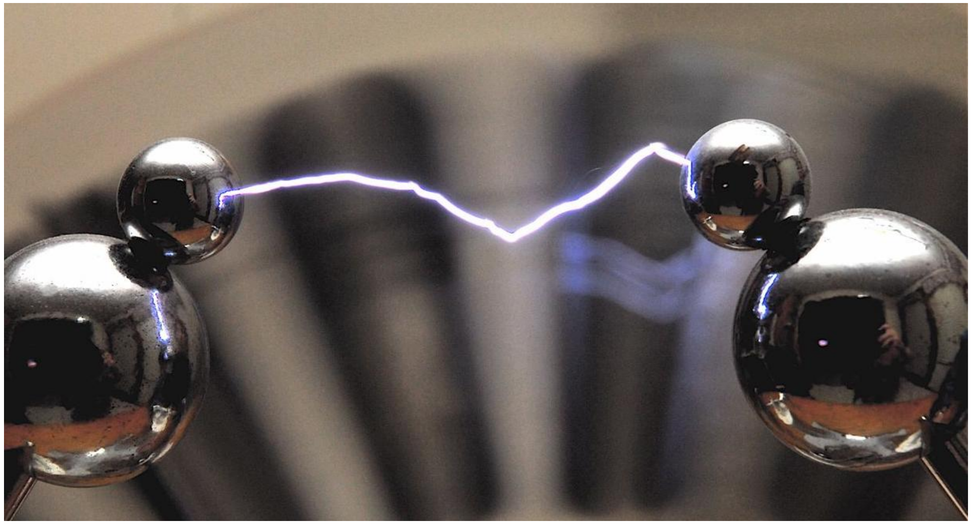
An intuitive understanding (qualitative): With the same distance and the same solid angle, the corresponding surface is large in the side with smaller curvature

Then, how about the negative curvature case?

2.1 Conductors

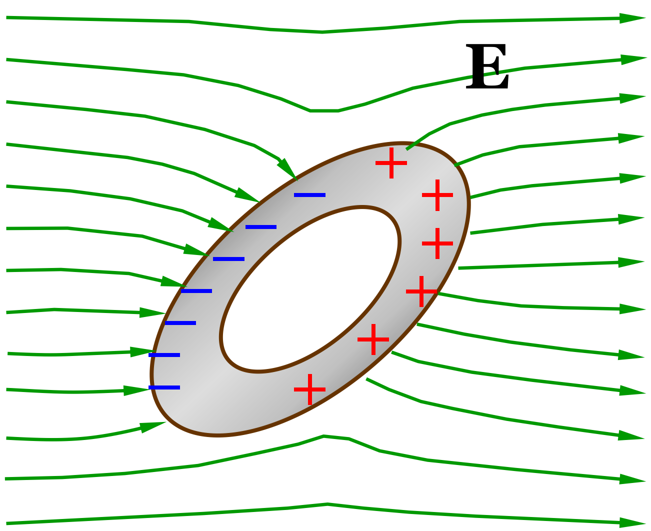


When the local electric field near the tip goes beyond a threshold (breakdown field), the corona discharge occurs.

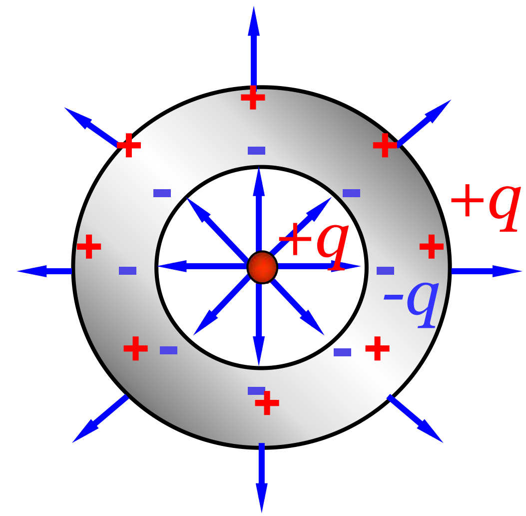


2.1 Conductors

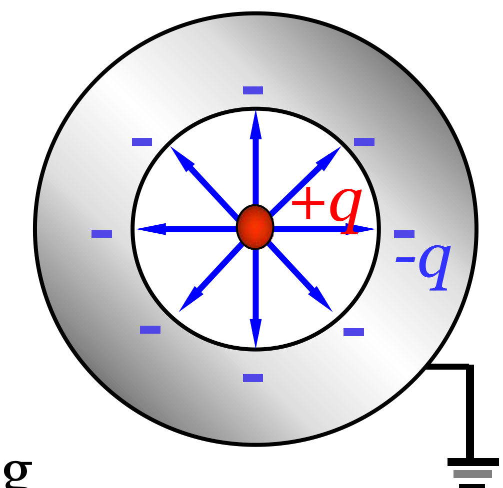
Electrostatic shielding



Outside field ---> zero inside



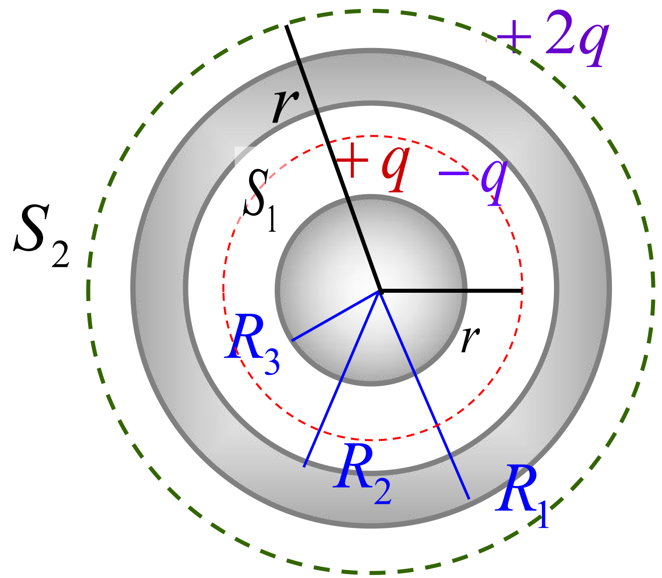
Inner charge ---> outside field



Grounding

2.1 Conductors

Example: an isolated hollow spherical shell (R_1: outside radius; R_2: inside radius) + a ball (R_3: radius), both of which are with charge q.



Spherical symmetric: $E(r)$, $U(r)$

1. $r<R_3$: $E=0$ (conductor)

U=a constant=$q/(4\pi\varepsilon_0 R_3)$??

2. $R_3<r<R_2$ $E(r)=q/(4\pi\varepsilon_0 r^2)$

$U(r)=q/(4\pi\varepsilon_0 r)+C$

$U(R_3)=q/(4\pi\varepsilon_0 R_3)+C=U_3$ ----> $C=U_3-q/(4\pi\varepsilon_0 R_3)$

$U(R_2)=q/(4\pi\varepsilon_0)(1/R_2-1/R_3)+U_3$

2.1 Conductors

$U(R_2)=q/(4\pi\varepsilon_0)(1/R_2-1/R_3)+U_3$

3. $R_2<r<R_1$ $E(r)=0$ (conductor)

$U(R_1)=U(R_2)$

4. $r>R_1$

$E(r)=2q/(4\pi\varepsilon_0 r^2)$

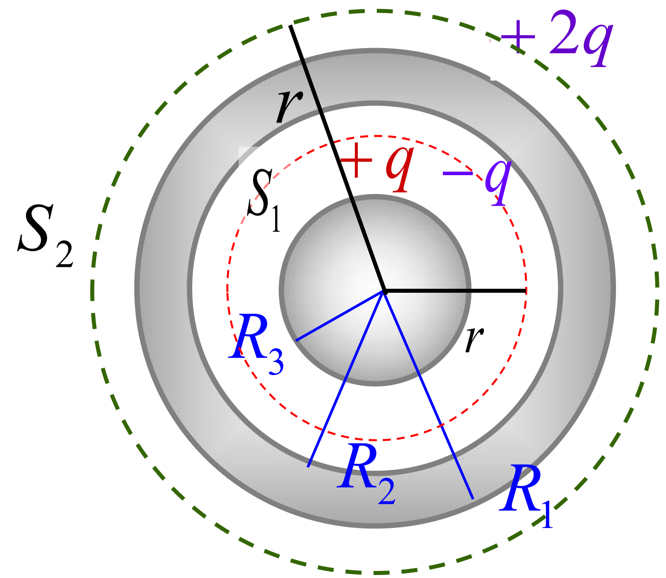
$U(r)=2q/(4\pi\varepsilon_0 r)$

$U(R_1)=2q/(4\pi\varepsilon_0 R_1)=U(R_2)$

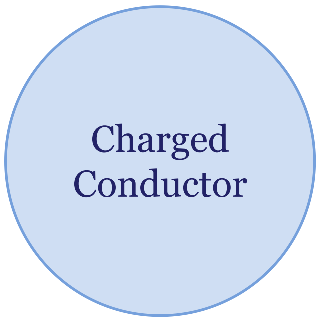
$2q/(4\pi\varepsilon_0 R_1)=q/(4\pi\varepsilon_0)(1/R_2-1/R_3)+U_3$

Then $U_3=q/(4\pi\varepsilon_0)(2/R_1-1/R_3+1/R_2)$

$\neq q/(4\pi\varepsilon_0 R_3)$



2.2 Capacitance & capacitors



$U \sim Q$ Linear superposition principle

e.g. $U=Q/(4\pi\varepsilon_0 R)$

Capacitance $C=Q/U=4\pi\varepsilon_0 R$

C: the charge needed to increase voltage

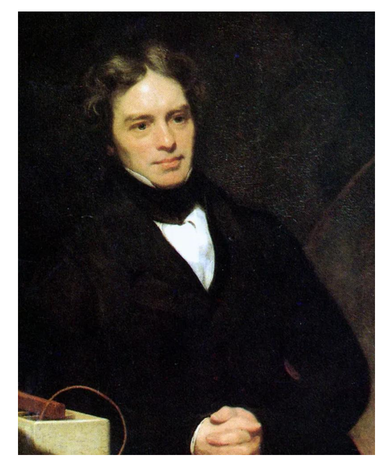
unit: F   1 F= 1 C/V

For isolated spherical conductor with 1 F

$R=1/4\pi\varepsilon_0=9\times10^9$ m = 1400 R_{earth}

Thus 1 F is too large in practice

Then 1 μF, 1 nF, 1 pF are more frequently used



Michael Faraday
1791-1867

2.2 Capacitance & capacitors

Two conductors with electrostatic shielding:

Then the electric field is only from the inner charge, independent on external field.

Case 1: a plane-parallel capacitor

$E=\sigma/\varepsilon_0$ $V=Ed$

$C=Q/V=\sigma S/(\sigma d/\varepsilon_0)=\varepsilon_0 S/d$

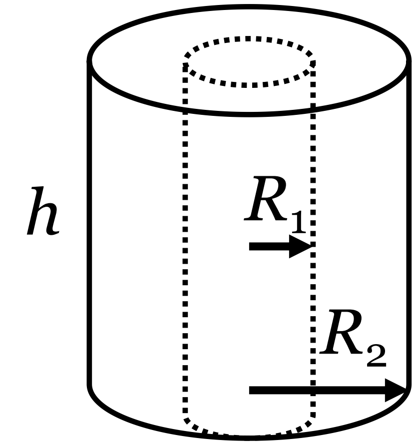
S=1 m^2 d=1 mm

$C=8.85\times10^{-12}\times1/0.001=8.85\times10^{-9}$ F

=8.85 nF

2.2 Capacitance & capacitors

Case 2: A coaxial cylindrical capacitor



$E=\eta/(2\pi\varepsilon_0 r)$

η: the line density of inner cylinder

$\Delta U=\int_{R_1}^{R_2} E dr=\eta/(2\pi\varepsilon_0)\ (\ln R_2-\ln R_1)$

$=\eta\ln(R_2/R_1)/(2\pi\varepsilon_0)$ $\qquad Q=\eta h$

$C=Q/\Delta U$

$=2\pi\varepsilon_0 h/\ln(R_2/R_1)$

e.g. $h=1$ m $\quad R_1\sim 0.5R_2$

$C=2\pi\times 8.85\times 10^{-12}/\ln(2)$

$=5.3\times 10^{-10}$ F$=0.53$ nF

In the limit of $R_1\sim R_2$

$\ln(R_2/R_1)=R_2/R_1-1$

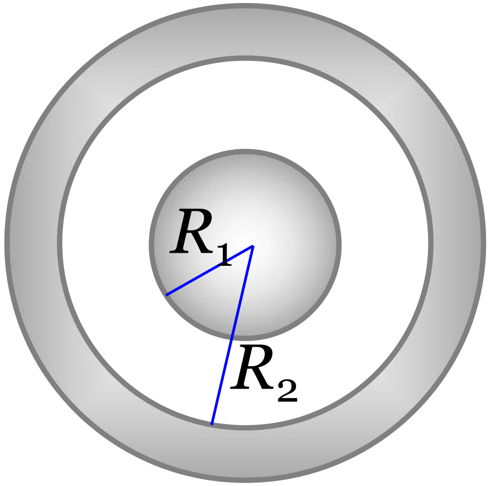
$C=2\pi\varepsilon_0 hR_1/(R_2-R_1)$

e.g. $R_1/(R_2-R_1)=1000$

$C=55.6$ nF

2.2 Capacitance & capacitors

Case 3: A homocentric spherical shells



$E = Q/(4\pi\varepsilon_0 r^2)$

$\Delta U = \int_{R1}^{R2} E dr = Q/(4\pi\varepsilon_0)\,(1/R_1 - 1/R_2)$

$C = Q/\Delta U$

$= 4\pi\varepsilon_0 R_1 R_2/(R_2 - R_1)$

e.g. $R_1 = 1$ m $R_2 \sim 1.1$ m

$C = 4\pi \times 8.85 \times 10^{-12} \times 1.1/0.1$

$= 1.2 \times 10^{-9}$ C $= 1.2$ nF

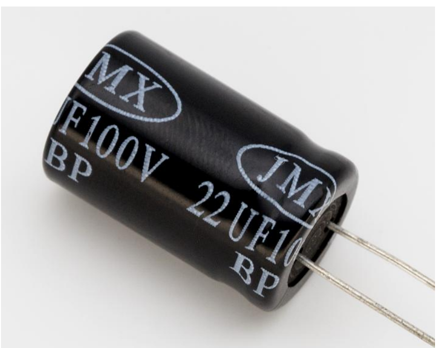
In summary, the general routes to enlarge C:

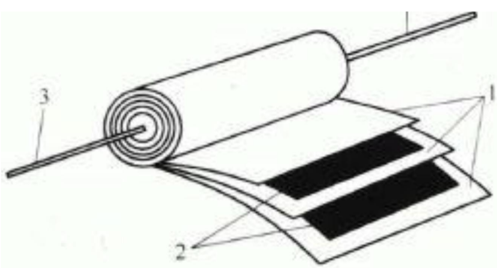
1. enlarge the charged surface
2. reduce the distance between charged surfaces

2.2 Capacitance & capacitors

Examples of different kind of capacitors

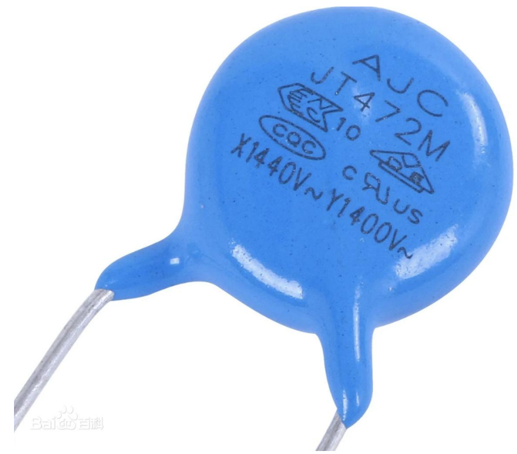
Paper capacitor

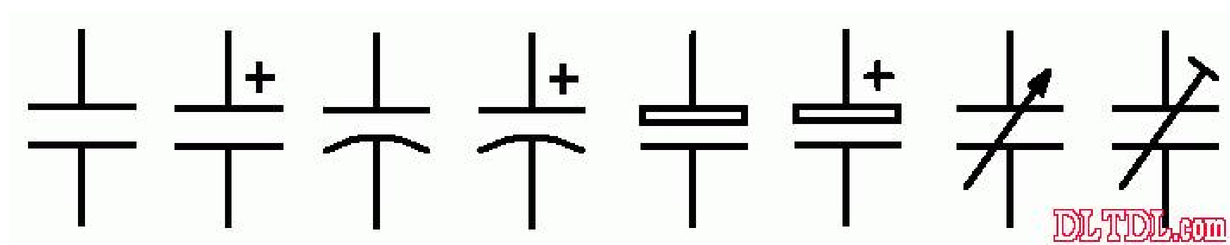




Aluminum electrolytic capacitor

Ceramic capacitor





2.2 Capacitance & capacitors

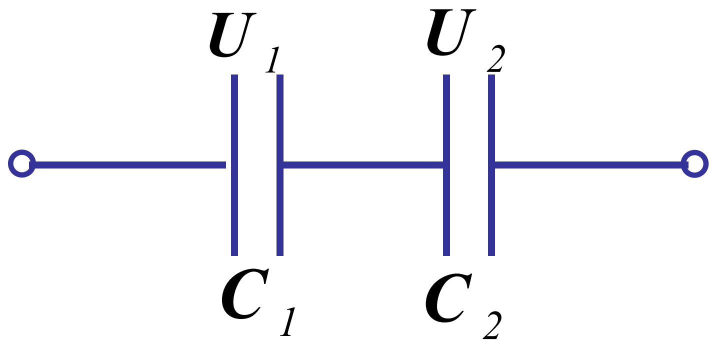
- Series connection

  $Q_1=Q_2=Q$

  $U=U_1+U_2$

  $C=Q/U=Q/(U_1+U_2)$

  $1/C=1/C_1+1/C_2$ C is smaller



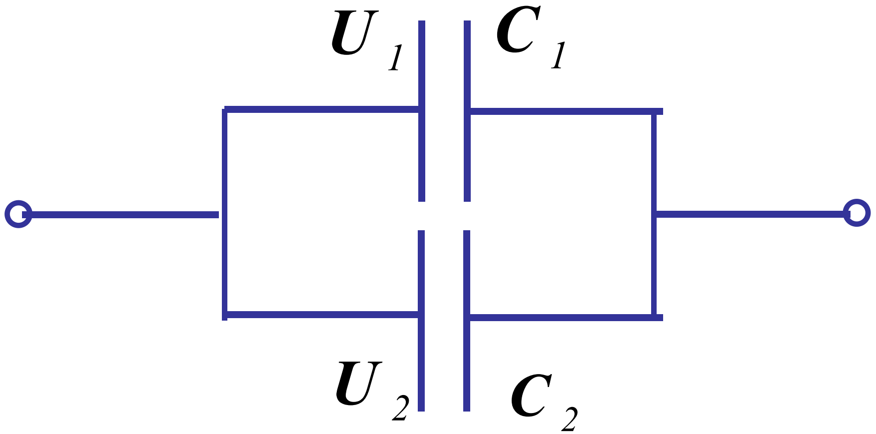
- Parallel connection

  $Q=Q_1+Q_2$

  $U=U_1=U_2$

  $C=Q/U=(Q_1+Q_2)/U$

  $C=C_1+C_2$ C is larger



Differnt from resistors!

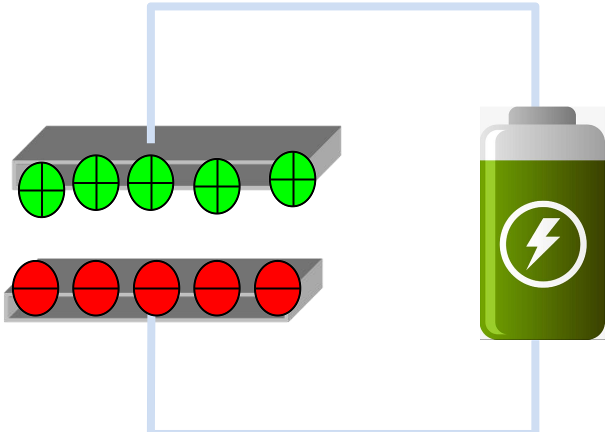
2.2 Capacitance & capacitors

The charging process:

$$dW_e = dq(U_+ - U_-) = U dq = q/C\, dq$$

$$W_e = \int dW_e = \int q/C\, dq$$

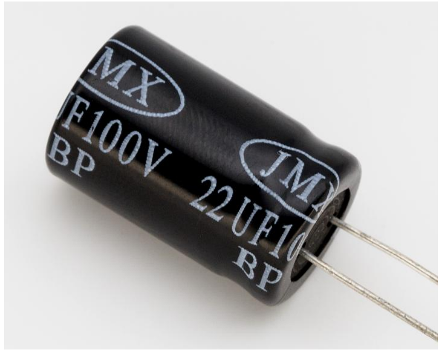
$$= Q^2/2C = CU^2/2 = QU/2$$



Example: how much energy can a paper capacitor store?

$$W_e = CU^2/2 = 22 \times 10^{-6} \times 100^2/2$$

$$= 0.11\ \text{J}$$



100 V, 22 μF

2.2 Capacitance & capacitors

AA chargeable battary 600 mAh 1.2 V

$W_e = UQ = 1.2 \times 0.6 \times 3600$

$= 2592$ J

AA alkaline battary 2500 mAh 1.5 V

$W_e = UQ = 1.5 \times 2.5 \times 3600$

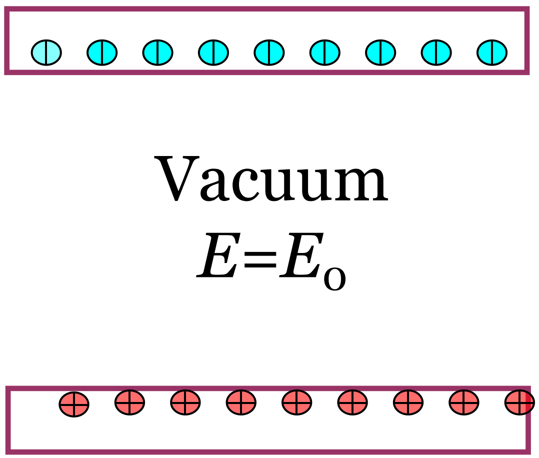
$= 1.35 \times 10^4$ J

Supercapacitor: 3000 F, 3 V

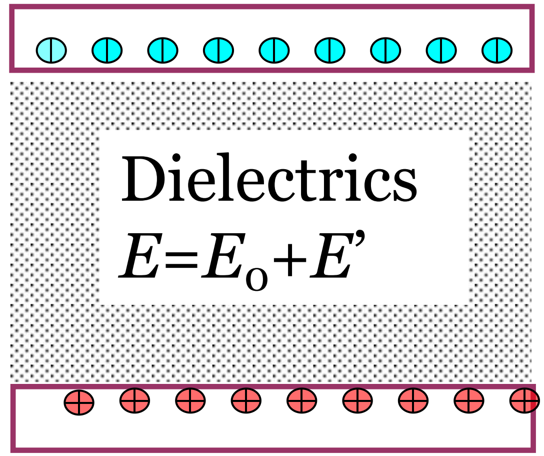
$W_e = CU^2/2 = 3 \times 10^3 \times 3^2/2$

$= 1.35 \times 10^4$ J $= 3.75$ Wh

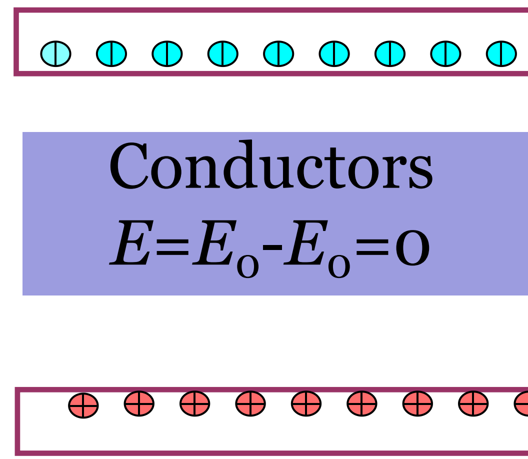
2.3 Dielectrics



no charge



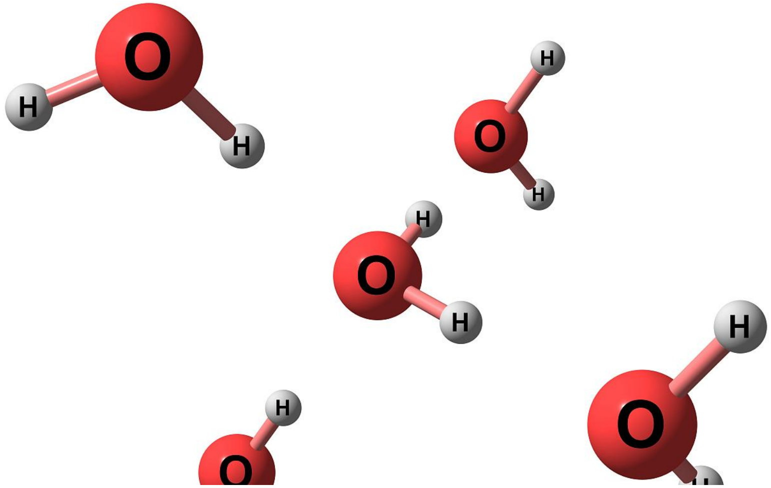
localized charges
(dipoles)



mobile charges
(carriers)

2.3 Dielectrics

a) Polar molecules



$\mathbf{p}_m \neq 0$ for each molecule

$\mathbf{P} = \sum \mathbf{p}_m = 0$ ($\mathbf{E} = 0$)

$\mathbf{P} = \sum \mathbf{p}_m \neq 0$ ($\mathbf{E} \neq 0$)

Electrical poling:

dipoles are aligned by electric field ---->

macroscopic electric polarization

$\mathbf{P} \sim \mathbf{E}$

2.3 Dielectrics

b) Nonpolar molecules

$\mathbf{p}_m$=0 for each molecule

$\mathbf{P}=\sum \mathbf{p}_m=0$ ($\mathbf{E}$=0)

$\mathbf{P}=\sum \mathbf{p}_m=0$ ($\mathbf{E}\neq 0$)?

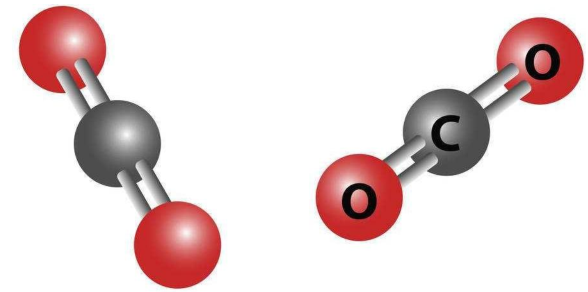
$\mathbf{P}=\sum \mathbf{p}_m\neq 0$ ($\mathbf{E}\neq 0$)

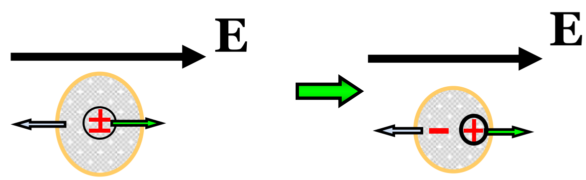
Electrical poling:

to creat dipoles by electric field ---->

macroscopic electric polarization

$\mathbf{P} \sim \mathbf{E}$





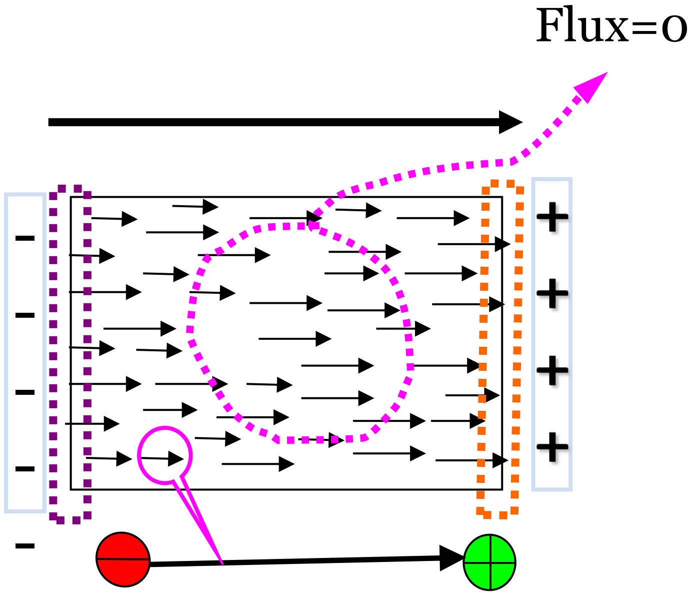
Which P is larger for these two cases? Why?

2.3 Dielectrics

Dielectrics under *E*-field

For a uniform dielectric medium,

- no net charge within the medium (macroscopically)
- net charges exist at the surface/interface (edge state)



Note: for a non-uniform dielectric medium,

- net charge can exist within the medium (macroscopically)

$$\nabla \cdot \mathbf{E} = \rho_e / \varepsilon_o$$

2.3 Dielectrics

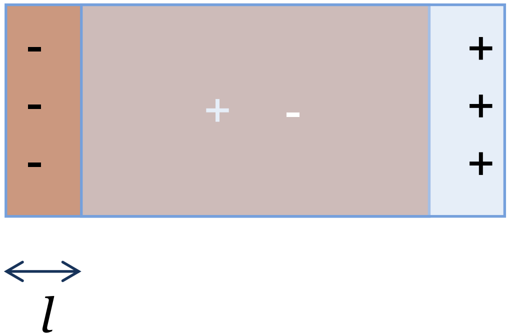
Electric polarization $\mathbf{P}=\sum_i \mathbf{p}_i/V$

unit: C/m^2 ~ surface charge density σ

ρ: volume density of dipole molecules

q: dipole charge

l: dipole distance



surface charge $Q^{+}=\rho\, q\, S\, l$

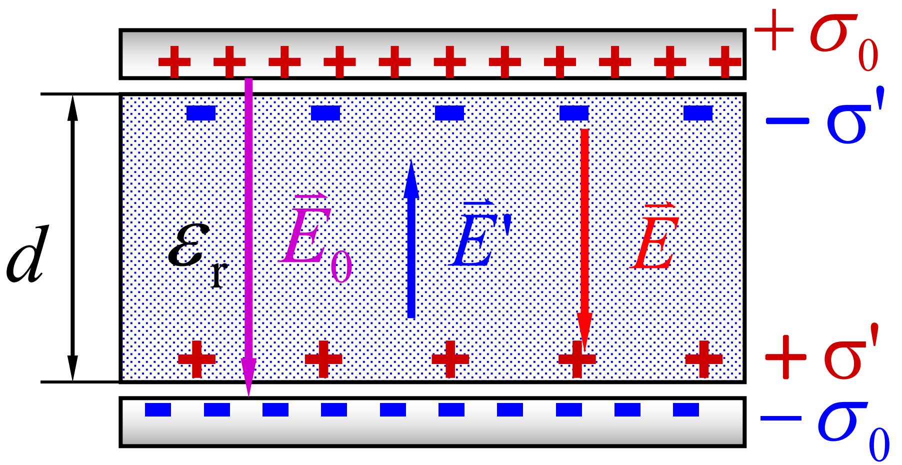
$\sigma'=Q^{+}/S=\rho\, q\, l=\rho p=P$

$\sigma'=\mathbf{P}\cdot\boldsymbol{e}_n$

not free charge, but bound charge

2.3 Dielectrics

Depolarizing field in the dielectric region



$E_o=\sigma_o/\varepsilon_o$

$E'=\sigma'/\varepsilon_o=P/\varepsilon_o$

$E=E_o-E'=E_o-P/\varepsilon_o=(\sigma_0-\sigma')/\varepsilon_o$

In linear dielectric medium

$P=\chi_e\varepsilon_o E$

χ_e: electric susceptibility

$E=E_o-E'=E_o-P/\varepsilon_o=E_o-\chi_e E$

$E=E_o/(1+\chi_e)$

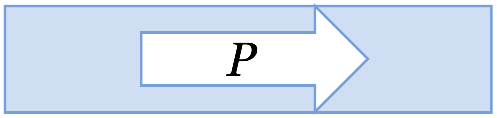
- $\sigma'=\sigma_0\chi_e/(1+\chi_e)$
- conducting limit:

  χ_e---->infinite
- vacuum limit

  χ_e---->0

2.3 Dielectrics

A uniformly polarized rod



$\sigma' = \mathbf{P} \cdot \mathbf{e}_n = P$, only exist at two ends

if $l^2 >> S$, these two ends can be considered as

two point charge $\pm PS$

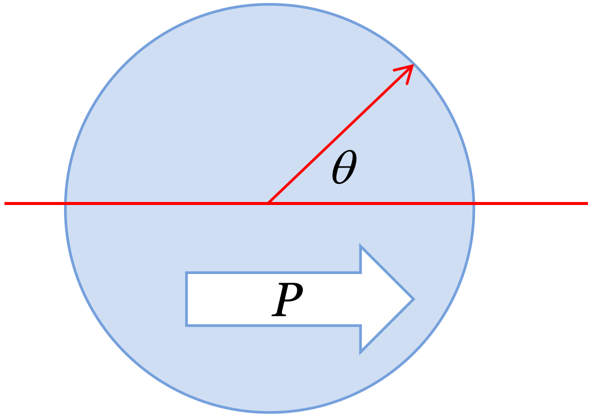
Then the depolarizing field at the center of rod

$E' = 2PS / [4\pi\varepsilon_0 (l/2)^2] = 2PS / (\pi\varepsilon_0 l^2)$

in the $l^2 >> S$ limit

$E' \text{----}> 0$

2.3 Dielectrics



A uniformly polarized ball

$\sigma' = \mathbf{P}\cdot\mathbf{e}_n = P\cos\theta$

$dS = R^2 \sin\theta d\theta d\varphi$

Then the depolarizing field at the center of ball

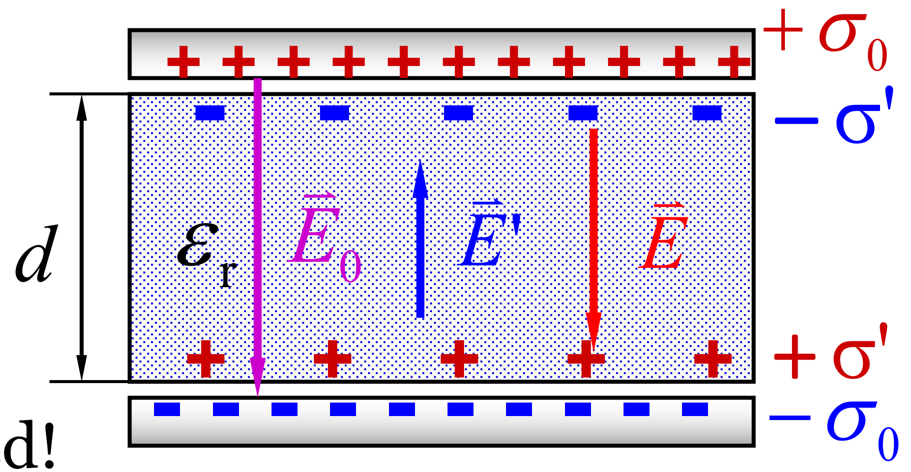
$dE' = dq'/(4\pi\varepsilon_0 R^2) = \sigma' dS/(4\pi\varepsilon_0 R^2) = \sigma' \sin\theta/(4\pi\varepsilon_0)\, d\theta d\varphi$

$dE_x' = -dE' \cos\theta = -P\sin\theta\cos^2\theta/(4\pi\varepsilon_0)\, d\theta d\varphi$

$E' = E_x' = \oiint dE_x' = P/(2\varepsilon_0) \int_0^{\pi} \cos^2\theta d\cos\theta = -P/(3\varepsilon_0)$

Depolarizing field depends on the geometry!

2.3 Dielectrics



$E=E_o/(1+\chi_e) < E_o$

$U=Ed<E_od=U_o$

$C=Q/U=(1+\chi_e)C_o$

The capacitance is increased!

By defining a **relative dielectric constant** $\varepsilon_r=1+\chi_e$

ε_r, χ_e: dimensionless quantities

Note: the linear isotropic *P*-*E* relation is only an approximation, which works in most cases.

It can be nonlinear and anisotropic, depends on materials and conditions.

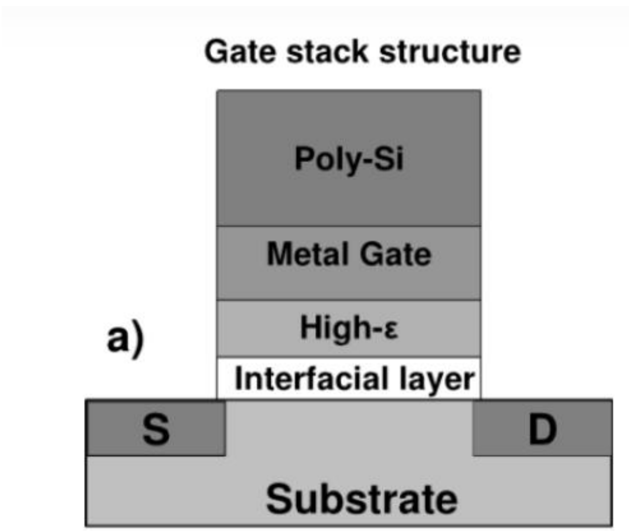
2.3 Dielectrics

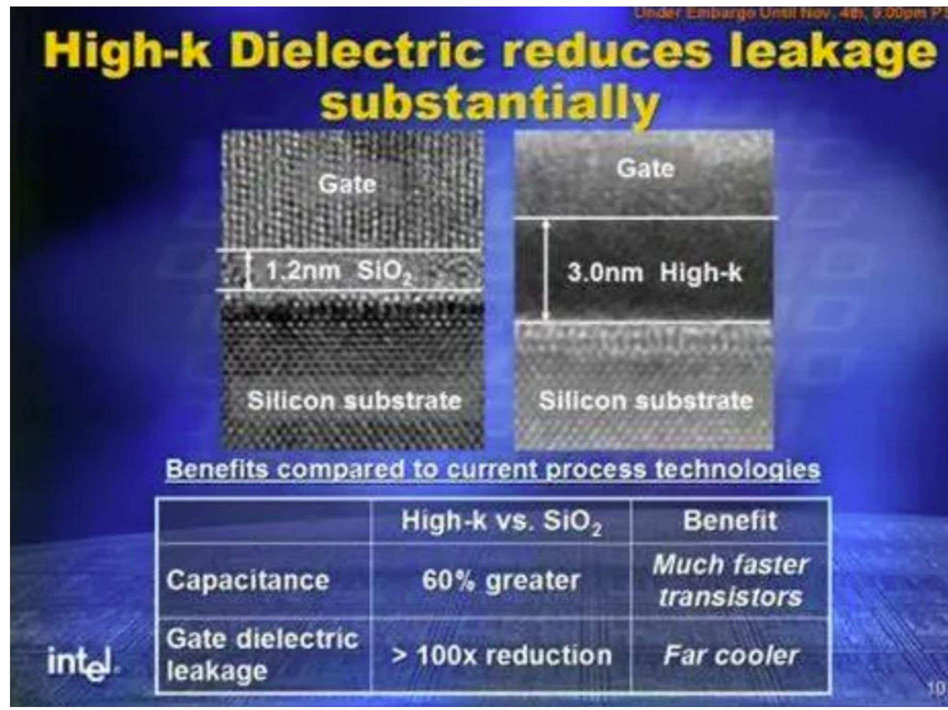
Dielectric constants of various substances

Substance	Conditions	Dielectric constant (κ)
Air	gas, 0 °C, 1 atm	1.00059
Methane, CH_4	gas, 0 °C, 1 atm	1.00088
Hydrogen chloride, HCl	gas, 0 °C, 1 atm	1.0046
Water, H_2O	gas, 110 °C, 1 atm	1.0126
	liquid, 20 °C	80.4
Benzene, C_6H_6	liquid, 20 °C	2.28
Methanol, CH_3OH	liquid, 20 °C	33.6
Ammonia, NH_3	liquid, −34 °C	22.6
Mineral oil	liquid, 20 °C	2.24
Sodium chloride, NaCl	solid, 20 °C	6.12
Sulfur, S	solid, 20 °C	4.0
Silicon, Si	solid, 20 °C	11.7
Polyethylene	solid, 20 °C	2.25–2.3
Porcelain	solid, 20 °C	6.0-8.0
Paraffin wax	solid, 20 °C	2.1–2.5
Pyrex glass 7070	solid, 20 °C	4.00

2.3 Dielectrics

Extension 1: high-k dielectric material: a vital key to continue the Moore's law





SiO_2 ε_r=3.9

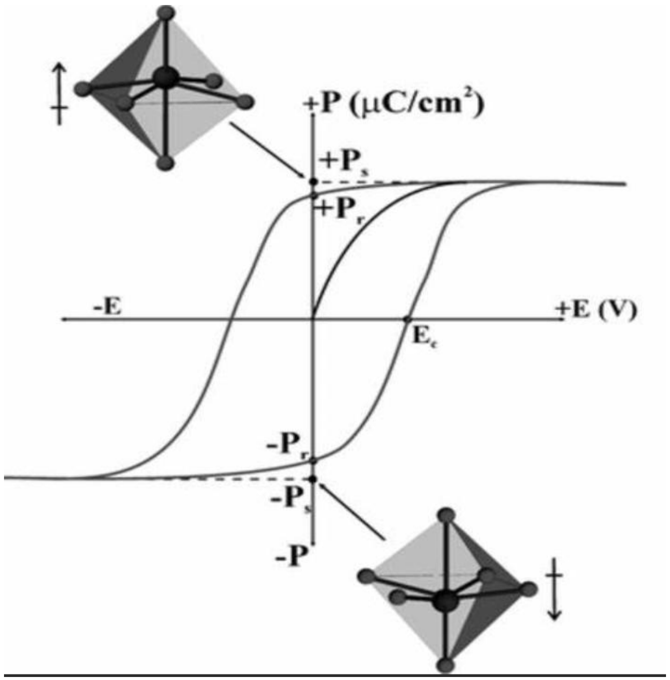
HfO_2 ε_r=20

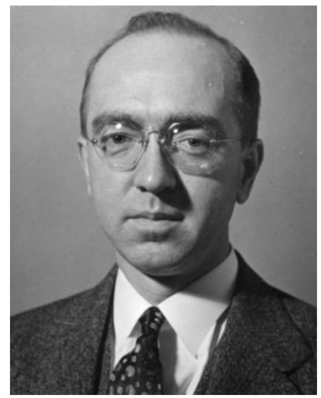
Supplemental reading:

HKMG(High-K 栅氧化物层 +Metal Gate)技术

2.3 Dielectrics

Extension 2: ferroelectrics





Joseph Valasek
(1897-1993)

《Physics Today》：
Because Valasek was a pioneer whose main work was done well before his field became popular, and because he was very quiet and modest and did not seek recognition and honor for his work, he never achieved the recognition he deserved."

P: nonlinear to *E*
non-volatile states
discovered in 1920

$BaTiO_3$, P_r=20 μC/cm²

$PbTiO_3$, P_r=80 μC/cm²

t-$BiFeO_3$, P_r=120 μC/cm²

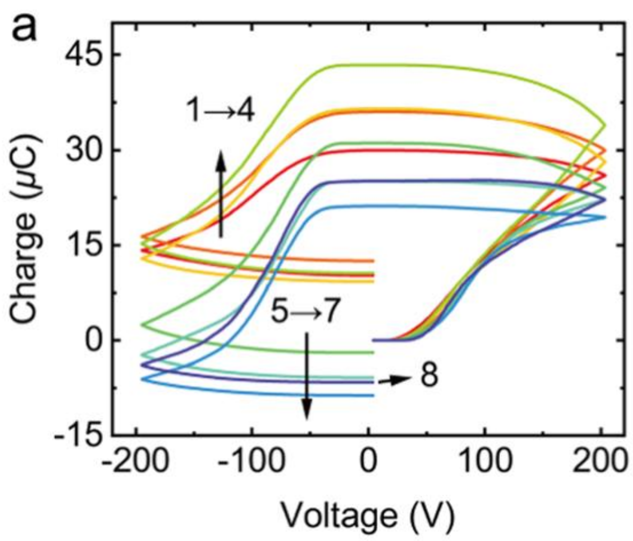
铁电百年诞辰，归来仍是少年

2.3 Dielectrics

JACS JOURNAL OF THE AMERICAN CHEMICAL SOCIETY
pubs.acs.org/JACS Article

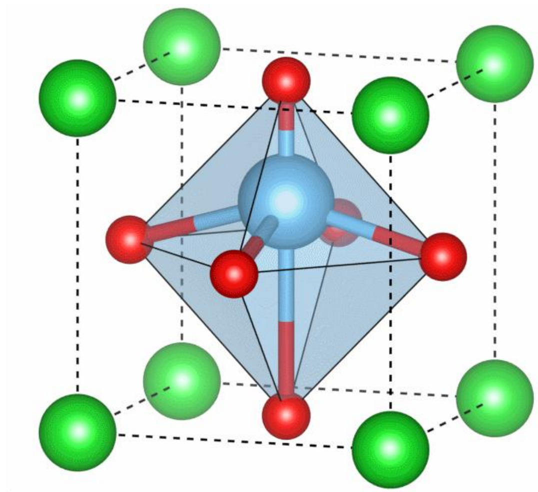
High, Multiple, and Nonvolatile Polarizations in Organic–Inorganic Hybrid $[(CH_3)_3(CH_2CH_2Cl)N]_2InCl_5 \cdot H_2O$ for Memcapacitor

Conventional ferroelectric materials based on displacive and order–disorder types generally have difficulty meeting these requirements due to their low polarization values (~150 μC/cm²) and persistent electrical hysteresis loops. In this study, we report a novel organic–inorganic hybrid $(CETM)_2InCl_5 \cdot H_2O$ (CETM =$(CH_3)_3(CH_2CH_2Cl)N$) exhibiting an intriguing polarization vs electric field (charge vs voltage) "hysteresis loop" and a record-high nonvolatile polarization over 30000 μC/cm² at room temperature.

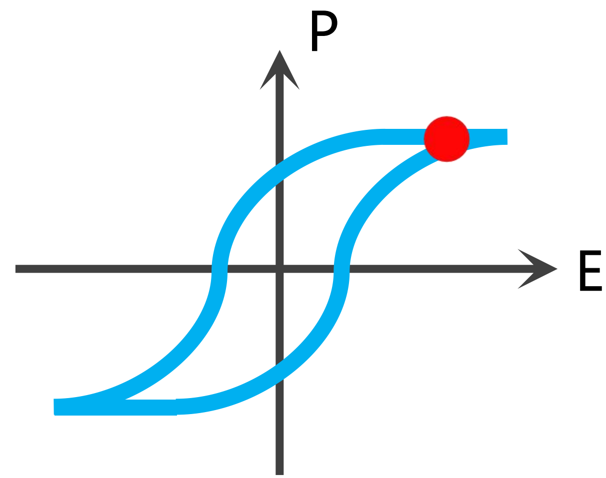


J F Scott: Ferroelectrics go bananas

铁电物理



铁电百年仍弥坚，
量子机制敢为先。
理论计算调极性，
磁电对偶续新篇。



第一代
罗息盐等
分子铁电体

第二代
钛酸钡、锆钛酸铅等
氧化物铁电体

第三代
磁性铁电体、
低维铁电体等

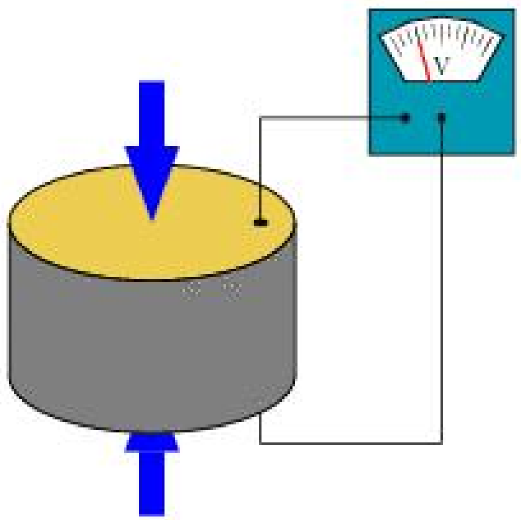
1920s　1940-1950s　2000s　2020s

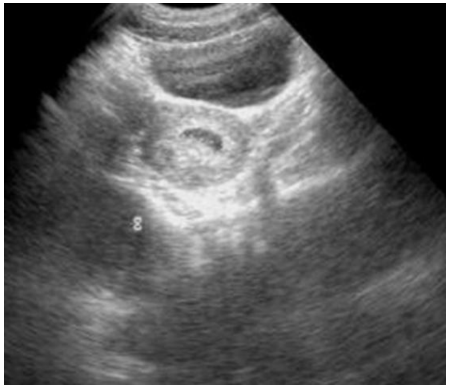
铁电经典理论

铁电量子理论

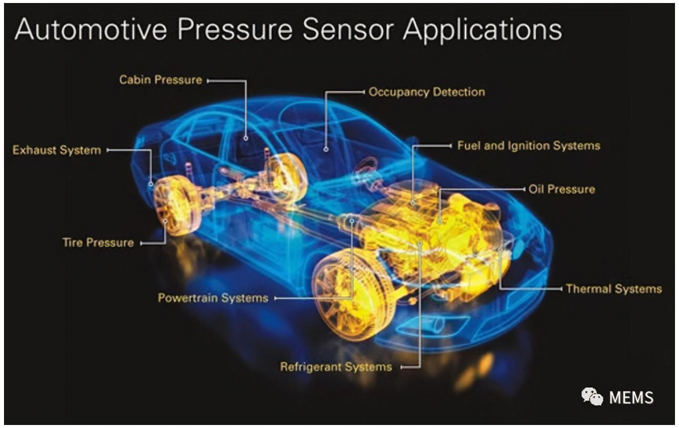
2.3 Dielectrics

Extension 3:
piezoelectricity





B-scan ultrasonography



sensors in industry

P changes upon pressure

Microelectromechanical Systems (MEMS)

biosensors

2.3 Dielectrics

Extension 4: electrets

N95 masks

Inventors:

蔡秉燚&刘朝宇(UTK)

electrets

Weakness:

Moisture

sensitive

For comparison:

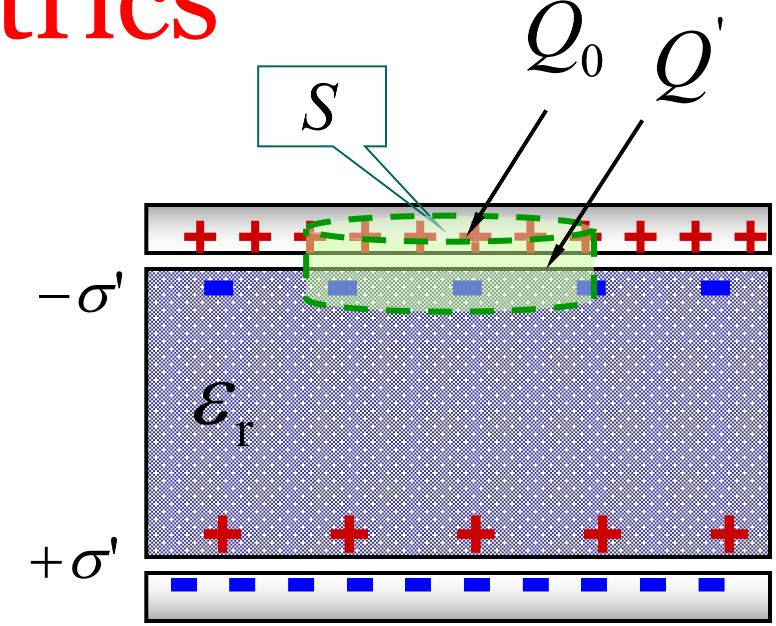
Graphene masks

Inventors:

孙立涛 (SEU)

ultra-large surface

2.3 Dielectrics



$\oint_S \mathbf{E}\cdot d\mathbf{S}=(Q_o-Q')/\varepsilon_o$

$Q'=\chi_e/(1+\chi_e)Q_o=(1-1/\varepsilon_r)Q_o$

$\oint_S \mathbf{E}\cdot d\mathbf{S}=Q_o/\varepsilon_o\varepsilon_r$

$\oint_S \varepsilon_o\varepsilon_r\mathbf{E}\cdot d\mathbf{S}=Q_o$

By defining a new vector field

$\mathbf{D}=\varepsilon_o\varepsilon_r\mathbf{E}=\varepsilon\mathbf{E}=\varepsilon_o\mathbf{E}+\mathbf{P}$

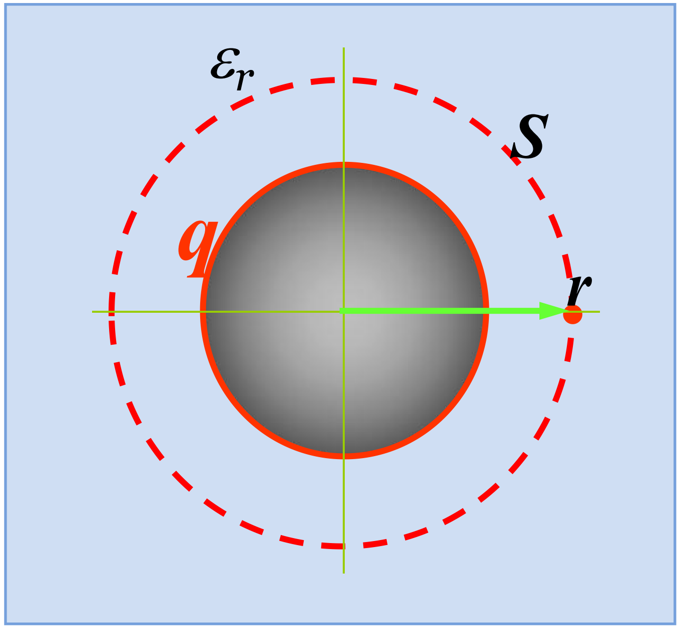
D: electric displacement vector; ε: permittivity of dielectric

Gauss's law in dielectrics: $\oint_S \mathbf{D}\cdot d\mathbf{S}=Q_o$ or $\nabla\cdot\mathbf{D}=\rho_o$

Only free carriers are involved. $\nabla\cdot\mathbf{P}=-\rho'$

2.3 Dielectrics

Electric field of a uniformly charged sphere/ball in uniform dielectric medium



outside:

$\oint_S \mathbf{D}\cdot d\mathbf{S}=Q_o$

$4\pi r^2 D=Q_o$

---> $D=Q_o/(4\pi r^2)$

$\mathbf{E}=\mathbf{D}/\varepsilon_o\varepsilon_r=Q_o\mathbf{e}_r/(4\pi\varepsilon_o\varepsilon_r r^2)$

$=\mathbf{E}_o/\varepsilon_r$

2.3 Dielectrics

Attention: whether $\mathbf{E}=\mathbf{E}_0/\varepsilon_r$ generally correct?

Answer: no!

The conditions:

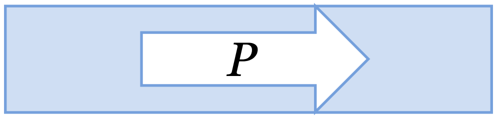
1) uniform dielectric medium in the whole space

2) uniform dielectric medium with isopotential surface

Example: A uniformly polarized rod

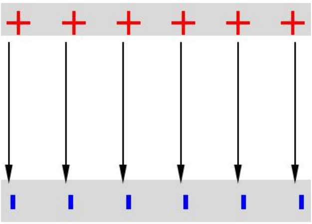
E'~0 in the middle point

$\mathbf{E}\approx\mathbf{E}_0\neq\mathbf{E}_0/\varepsilon_r$



2.4 Energy of electrostatic fields

The electrostatic energy of a plane-parallel capacitor:



$W_e = QU/2 = CU^2/2$

$= \varepsilon S/d\,(Ed)^2/2 = \varepsilon E^2 V/2$

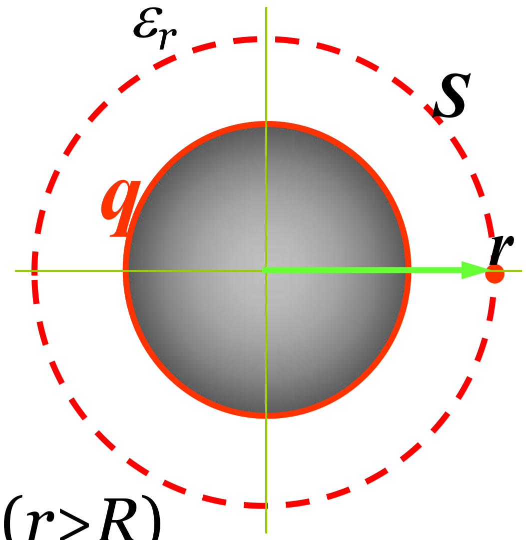
Energy density

$w_e = W_e/V = \varepsilon E^2/2 = DE/2$

$W_\mathrm{e} = \int w_e dV$ generally works beyond the plane-parallel capacitor

Philosophy: the energy is carried by the field, instead of charge!

2.4 Energy of electrostatic fields



From the viewpoint of field:

Case 1: a uniformly charged sphere

$W_e = \iiint w_e dV = \iiint \varepsilon_0 E^2 dV/2$

$= \varepsilon_0/2 \iiint [q/(4\pi\varepsilon_0 r^2)]^2 dV$ (outside)

$= q^2/(32\pi^2\varepsilon_0) \iiint 1/r^4 dV = q^2/(8\pi\varepsilon_0) \int 1/r^2 dr$ $(r>R)$

$= q^2/(8\pi\varepsilon_0 R)$ equal to the value calculated from charge

In dielectric medium,

$W_e = \iiint w_e dV = \iiint DEdV/2 = q^2/(8\pi\varepsilon_0\varepsilon_r R)$

Chapter 2: Homework

1. 9-2, 9-3, 9-5, 9-11, 9-14
2. Additional homework: to calculate the field energy of a uniformly charged ball
3. Deadline: Tuesday, May 7