Stimulation of a single lane traffic flow

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Abstract
A single-lane traffic flow model stimulation based on the optimal velocity theory is adopted to investigate the spontaneous formation of traffic jam in a continuous flow. With reference to the judgment of the stability, a two-phase flow is defined.
keywords: optimal velocity model; traffic flow

I. Introduction

Since motor vehicles have been massively used, great numbers of problems have appeared, e.g. traffic jam. Some physicists and mathematicians have involved in such traffic problems. In 1950s, physicist James Lighthill and applied mathematician Gerald Whitham [3] established a model (Lighthill-Whitham mode) where traffic low on a highway was compared to water in the tube from a perspective of hydrodynamics. Further in 1990s, the three-phase traffic theory was developed by Boris Kerner, who illustrated traffic flow in a diagram of phase transition with phases defined in traffic flow as free flow(F), synchronized flow(S) and wide moving jam (J).The theory above is typically a macroscopic model. Still many microscopic traffic flow models which focus on the behaviour of individuals have been developed, for example, cellular automaton theory, optimal velocity theory (Bando et al) which is adopted in the article and will be introduced later.

II. Model

The phenomenon has sometimes been found that vehicles get trapped in a traffic jam in a continuous traffic flow without any observable incidents, which is considered as a local cluster. Any infinitesimal perturbation could cause the formation of the spontaneous formation of the jam which indicates that the traffic flow should be in an unstable equilibrium.

The phenomenon is investigated under the scheme of the optimal velocity model by Bando et al. where the dynamics of the traffic flow is described by Eq.(1)

$$\ddot{x}_n = a[V(b) - \dot{x}_n] \tag{1}$$

The variable x_n represents the location of the n_{th} vehicle on the road. The coefficient a is a constant (called the sensitivity of the system). V is a function dependent on $b = \Delta x_n = x_{n+1} - x_n$ (the distance apart from the front vehicle), called the optimal velocity The function V should be (i) monotonically increasing (ii) up-bounded (iii) nonnegative from the assumptions or fact that (a) a driver tends to speed up and shorten the distance apart from the front vehicle (b) The velocity must have maximum value in reality, e.g. the speed limit in different areas. (c) Backing is usually forbidden in a continuous one-way traffic flow.

The expression of function $V(\Delta x)$:

$$V(b) = \tanh(b-2) + \tanh 2$$

For simplicity, the single-lane simulation was adopted. The density of vehicles is defined as N/L with the total car number denoted by N and length of the road, L. And a boundary condition that N_{th} vehicle follows the first one in the flow is set, which makes it actually a closed loop.

III. Result:

We may soon get a solution of Eq.(1), which is a steady flow, expressed in Eq.(2). It also satisfies the realistic situation.

$$x_n = vt + c_n \quad (2)$$

In order to study the stability of the steady state flow, we add a small perturbation to x_n. And it proves [1] that the stability of the solution is decided by Eq.(3)

Stable: $f < a/2$

Unstable: $f > a/2$

and $f = V'(b)$ (3)

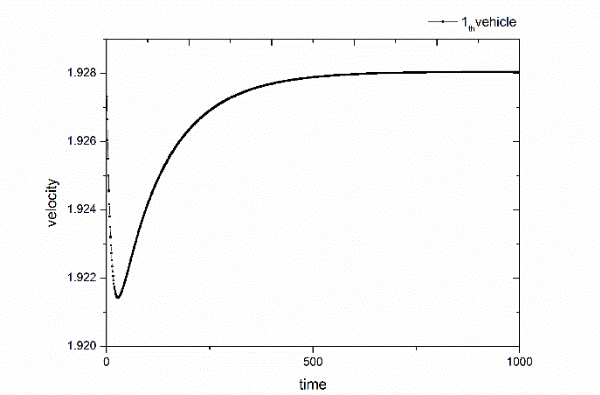


FIG.1 Time behaviour of velocity of the first vehicle

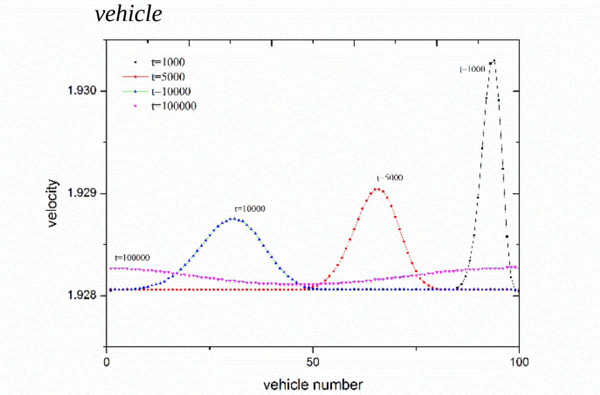


FIG.2 Velocity configuration in the stable situation at t=1000, 5000, 10000, 100000. A decay of amplitude is displayed.

The result of numerical simulation in two different situations is presented below. For convenience, the coefficient a was chosen to be 1. The time unit was adopted as dt=0.1 in the simulation.

In the stable situation:

$$N = 100, L = 400; b = L/N = 4$$
$$f = V'(b) = 1/\cosh^2(b-2) = 0.0707 < 0.5$$

In this situation, the optimal velocity $V_B = 1.9281$. A small subtle decrease in velocity of some vehicles can be observed, after which, however, the velocity returns to the origin value V_B. The velocity distribution in vehicles is depicted in Fig.(2). The initial perturbation caused a time-decayed wave propagating upstream and the effect won't be last for a long time, which also indicates such state is stable.

while in the unstable situation:

$$N = 100, L = 200; b = L/N = 2$$
$$f = 1 > 0.5$$

The origin is $V_B = 0.9642$. A more wide and frequent variance can be easily found after a short time in Fig.(3) and the upper bond and lower bond was often reached. The initial perturbation evolved into great fluctuation and resulted in this frequent local cluster (see Fig.4).

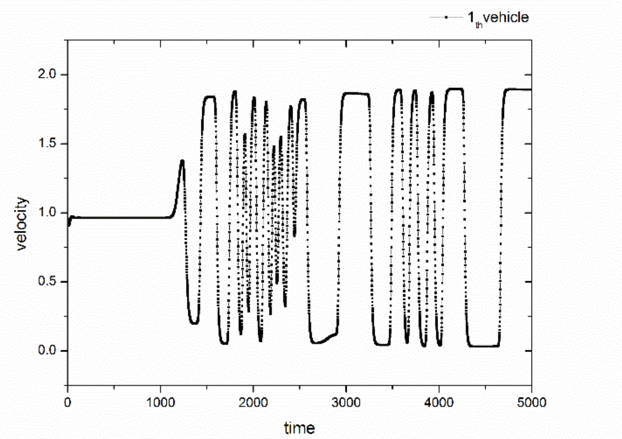
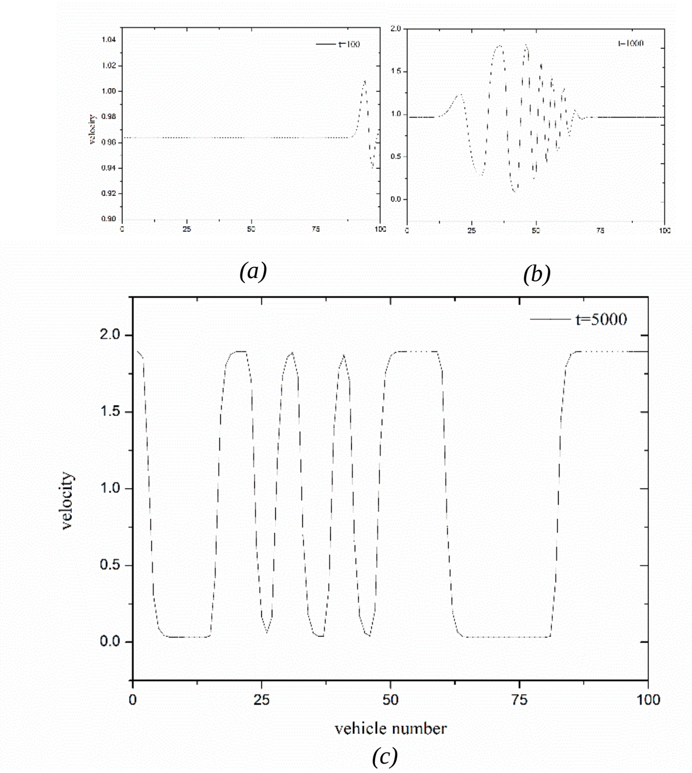


FIG.3 Velocity configuration at t=1000, 5000, 10000, 100000. A decay of amplitude is displayed. The local jam occurring at v=0. The traffic flow is in unstable equilibrium.



(a) (b) (c)

FIG.4 (a), (b), (c) Velocity configuration in the unstable situation at t=100, 1000, 5000. Subtle perturbation grows into "fierce oscillation"

In FIG.5 The occurrence and propagation of local clusters is very obvious. the dark region indicates a high local density, of course, where a jam tends to happen.

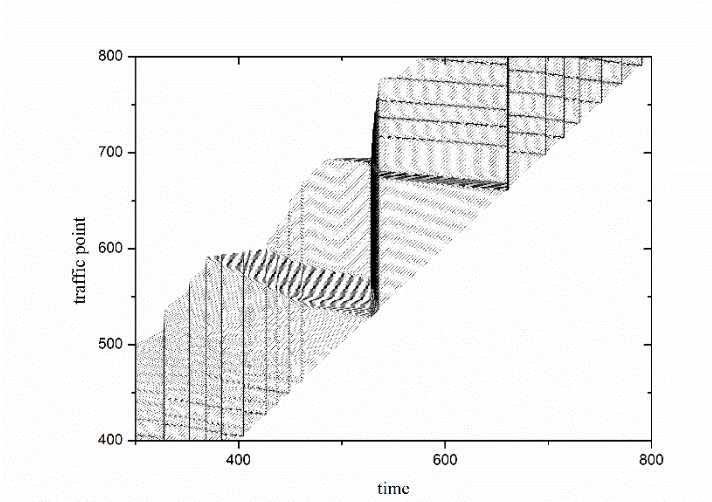


FIG.4 (a), (b), (c) Time evolution of vehicles in the unstable state with N=200, L=400;b=L/N=2

IV.Discussion

After numerical simulation, we now recall the judgment of stability in Eq.(1).

$$f < a/2(stable),$$
$$V'(b) < a/2$$

In this model, we have:

$$a > 2/\cosh^2(b-2)$$

(we usually take $a > 0, b > 0$)

It provides us with an equation of two parametres $a \& b$ with which we can define a two-phase transition in this simplified single-lane traffic model. See in FIG.6. The curve divided the plane into two parts. The part below the curve is an unstable state where traffic jam (or, in reality, accidents) tends to happen while the other part is denoted by a stable state, where traffic flow inclines to form a steady flow.

In reality, the sensitivity a of the system should be influenced by many environmental variables, like time (drivers' states), driving behavior, weather, etc which is very difficult to control, even precisely measured by experiments. However, in FIG.6 we find in the area where $b > 5$ there's always a stable state. And the parameter b is much easier to regulate, for example, by setting a minimum b in some sections of a road. And it may offer a way of shunting in arterial roads.

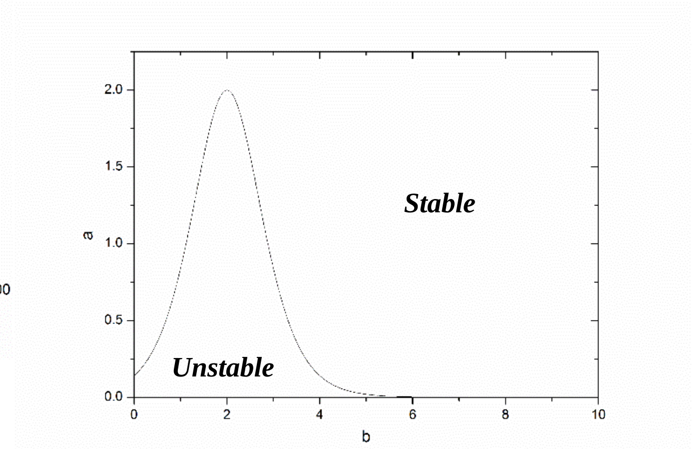


FIG.6 two phases defined in the sing-lane traffic flow model

However, the values of parametres was chosen here in consideration of the convenience of analysis. Whether the corresponding values remains suitable and practical hasn’t been studied. As has been mentioned before, it’s a greatly simplified model of a single lane, thus, ignoring the effect of change lanes, which will improve the complexity of the system. For example, in Boris Kerner’ s three phase theory, the appearance of “synchronized flow” phase describes the phenomenon that a tendency towards synchronization of vehicle speeds across different lanes in a continuous flow. Besides, as a microscopic model, some more details need to be taken into consideration.

References

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